been that of the great feast of the Niceteria in honour of Minerva's contest with Neptune for the protectorate of Athens, and the other, the annual celebration of the Marathon victory.

As respects the year 447, which is one year earlier than the supposed commencement of the Parthenon, it seems appropriate because at that time Pericles would have been supreme, and it is likely that a temple of such sanctity as the Erechtheum would have called for his earliest attention. It is true that the temple remained long unfinished, but of the causes of this delay we are ignorant. The connection however of the orientation with the feasts above mentioned would have been exactly the same if the date had been 428.

The apparent discrepance between the orientation date (as respect the day of the month) in the case of the Temples of Jupiter Olympius, and that which is supposed by Mommsen to have been the day of the celebration of the great feast to the supreme god (namely Munychion 19, the tenth month of the year, which in a general way corresponded with April), whereas the orientation dates give for the earlier temple March 30–31, and for the later March 27, is explained by the possibilities of the Metonic cycle, for when the Attic year began as it would in its course on July 11, the 19th Munychion would agree with March 30, or if July 8 with the 27th of March.

"Collimator Magnets and the Determination of the Earth's Horizontal Magnetic Force." By C. Chree, Sc.D., LL.D., F.R.S., Superintendent of the Kew Observatory. Communicated by the Kew Observatory Committee of the Royal Society. Received May 31,—Read June 15, 1899.

Contents.

§ 1. The present paper deals with magnets employed in measuring declination and horizontal force. More than 100 collimator magnets of English make have been examined at Kew Observatory, and the record of the results forms probably a unique mine of information. So far as I know, the only use hitherto made of this has been in the compilation of a statistical paper on the mean and extreme values of the temperature and induction coefficients by the late Mr. G. M. Whipple.*

The examination of a collimator magnet at Kew Observatory consists mainly in the determination of certain "constants." These are the "temperature coefficients" q and q', the "induction coefficient" μ, and the "moment of inertia" K.

The values found for these "constants" are utilised in the construction of tables, intended for reducing the observations of horizontal force. After the tables are constructed one or two observations are made. Their primary object is to ensure that the application of the tables leads to satisfactory results, and that there are no instrumental defects; but incidentally they afford the means of determining the magnetic moment, m, of the collimator magnet, and also the value of a "constant," P, appearing in the expression

$$2\,mm''r^{-3}(1+\mathrm{P}r^{-2})$$

for the couple exerted by the collimator magnet on an auxiliary magnet, moment m'', at distance r. The investigations described in the present paper have been prosecuted at intervals during the last five years, as the pressure of other work allowed. Their object has been twofold, 1° to find out whether any relationships exist between the several constants, and 2° to ascertain where our present knowledge wants amplification, and where the present tests are least satisfactory.

I shall first explain the real significance of the "constants," and describe briefly the method of determining them.

§ 2. *Temperature Coefficients.*—It is assumed that the magnetic moment, m, at a temperature of $t°$ C., the magnet being free from external force, is connected with the moment m at 0° C. by the relation

$$m'/m = 1 - qt - q't^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1),$$

where q and q' are absolute constants for the magnet concerned. If (1)

* 'Roy. Soc. Proc.,' vol. 26, pp. 218—222, 1877.

be taken for granted, q and q' can be determined by observing m' at any three convenient temperatures. These temperatures, as the experiment is conducted at Kew Observatory, lie usually within a degree or two of 0°, 18°, and 36° C. Magnets, however, destined for Arctic work are exposed to a temperature below 0° C., while magnets destined for tropical regions are exposed to a temperature over 40° C.

The changes of temperature are made rapidly by introducing hotter or colder water into a wooden box containing the collimator magnet. Changes in the moment of this magnet, accompanying observed changes in the temperature of the water, are deduced from the variations in azimuth of an auxiliary magnet, freely suspended at a fixed distance from the deflecting magnet. In a single experiment, the cycle of temperature "hot," "mean," "cold" is repeated three times, and the mean of the three observations at each temperature is used in the final calculation. It is customary to have two completely independent experiments on different days, and to take the arithmetic mean of the values deduced for q and q' on the two occasions. The exact times are noted at which the several readings are taken, and suitable corrections are applied from the readings of the magnetic curves for variations in the horizontal force and declination.

§ 3. *Induction Coefficient.*—This is denoted by μ, and really means the temporary change in the magnetic moment of the collimator magnet due to unit change in the field (parallel to the magnet's length), it being assumed that the relation between temporary moment and strength of field is linear.

The experiment* consists in observing the angles through which an auxiliary magnet is deflected out of the magnetic meridian by the collimator magnet, the latter being vertical, with its north pole alternately up and down. The vertical plane through the centres of the deflecting and deflected magnets is perpendicular to the latter's axis, but the centres of the magnets are *not* in the same horizontal plane.

The change in the inducing field being double the intensity of the vertical force at Kew is nearly 0·9 C.G.S. unit. As a rule, only one complete experiment is made, but this involves inverting and reinverting the magnet several times. Originally μ was measured in British units, so that conversion into C.G.S. units was necessary in many cases.

§ 4. *Moment of Inertia.*—This means the moment of inertia of the magnet and all its appendages, when at a temperature of 0° C., about the suspending fibre. A collimator magnet is a hollow steel cylinder, about $9\frac{1}{3}$ cm. long and 1 cm. in external diameter, with screws cut on its inner surface at both ends. The appendages consist of two small cells, one holding a lens the other a glass scale, screwed into the

* A special apparatus is employed whose description would occupy undue space. The method is practically that described on pp. 151–3 of Lamont's 'Handbuch des Erdmagnetismus.'

ends of the magnet, and of a brass stirrup arrangement which carries the magnet and affords the means of supporting, parallel to it, an auxiliary solid brass cylinder. This brass cylinder is a regular geometrical object, whose moment of inertia can be calculated from its weight, length, and diameter. The actual inertia experiment consists in observing the times of vibration of the magnet, under the earth's horizontal force, when the auxiliary bar is in the stirrup, and when it is removed. To reduce the possible effects of variation in force or temperature, four complete series of vibrations are made, the first and fourth without, the second and third with the auxiliary cylinder. Allowance is made for the departure of the mean temperature from 0° C. It is customary to make two independent determinations of the moment of inertia, usually on different days, and in the event of serious discrepancy a third experiment is made. In all the older experiments conversion from British to C.G.S. units was necessary.

§ 5. *Coefficient* P.—The meaning of this has been already explained generally. I need only add that in the deflection experiment the axes of the two magnets are perpendicular, and the centre of the deflected or, as it is called, "mirror" magnet lies on the axis produced of the collimator magnet. The general expression for the couple exerted in such a symmetrical position, the centres of the magnets being at distance r, is

$$2mm'r^{-3}(1+\mathrm{P}r^{-2}+\mathrm{Q}r^{-4}+\ldots\ldots),$$

where P, Q,......are constants, whose values depend on both the deflecting and deflected magnets. As r increases, the terms involving the higher negative powers of r tend to vanish relative to the first term; and in the present case it is assumed that the term involving P is the last that need be retained. When this is true we can determine P by comparing the couples answering to any two different values of r. The distances originally adopted at Kew Observatory when British units were employed were 1 foot and 1·3 feet; the distances now in use are 30 cm. and 40 cm. For several reasons, I have recorded the value not of P but of P/r^2 at 30 cm., allowing in cases where British units had been used for the small difference between 30 cm. and 1 foot.

§ 6. The number of makers of collimator magnets is not large, and the differences between the patterns are mostly small. Still it seemed undesirable to wholly disregard the differences that unquestionably exist. Accordingly, in summarizing the results contained in the Observatory records, I have divided the magnets into six groups, distinguished by the letters A to F. Four of the groups, A, B, C, and E, contain magnets from one maker only, and are presumably fairly homogeneous. Group D is the most miscellaneous, containing magnets by three if not four makers; group F contains magnets from two makers only.

Table I gives the mean values of the various quantities specified for the several groups; while Table II gives the extreme values.

Table I.—Mean Values.

Group.	Number in group.	π^2K.	m.	$10^6 q$.	$10^8 q'$.	μ.	$10^5 \times P/r^2$ at 30 cm. When +. Number.	When +. Mean value.	When −. Number.	When −. Mean value.
A	7	3532	921	328	120	6·70	4	281	3	293
B	12	2767	672	257	119	6·03	0	—	9	430
C	82	2755	873	349	144	5·64	40	843	31	354
D	12	2657	854	302	134	6·43	4	918	7	290
E	10	2520	837	349	156	7·56	10	655	0	—
F	7	1667	633	350	139	3·71	4	545	0	—
Totals and means	130	2711	840	335	140	5·85	62	762	50	361

Table II.—Extreme Values.

Group.	π^2K.	m.	$10^6 q$.	$10^8 q'$.	μ.	$10^5 \times P/r^2$ at 30 cm. When +.	When −
A max.	3741	1142	510	185	8·27	374	447
A min.	3037	653	180	23	4·36	64	353
B max.	3184	799	359	224	7·07	—	688
B min.	2334	481	211	14	5·31	—	121
C max.	3072	1115	764	312	6·88	2529	1438
C min.	2483	462	167	19	4·19	30	10
D max.	2825	1197	687	261	11·89	1775	619
D min.	2352	505	195	51	4·23	520	122
E max.	2658	906	370	332	7·79	1188	—
E min.	2422	723	321	82	7·02	173	—
F max.	1758	777	604	339	4·81	693	—
F min.	1600	451	273	35	2·57	383	—

Data were wanting for P in the case of eighteen out of the 130 collimator magnets, and there were no data for K in the case of three magnets, all of group C. In these twenty-one cases m was calculated from the

recorded value of the angle through which the magnet deflected an auxiliary magnet at a distance of 1 foot, that being a quantity determined in the induction experiment. Old magnets, when remagnetized and retested, have been treated as new magnets and counted separately.

§ 7. The discussion of Tables I and II would have been much simplified if one had possessed a record of the weight and dimensions of the collimator magnets themselves. It is not, however, customary to measure these quantities at the Observatory, as they in no way enter into the tables used in reducing the observations.

The only data at my disposal, bearing directly on these points, are the results of special measurements recently made on six magnets, three belonging to group C, and three to group E.

The results were as follows, the numbering being purely arbitrary; the lengths are in centimetres, the weights in grams:—

Table III.—Dimensions and Weights of specimen Magnets.

Group.	Magnet.	Length.	External diameter.	Internal diameter at end.	Weight magnet.	Weight appendages.
		cm.	cm.	cm.	grams.	grams.
C	i	9·36	1·00	0·81	27·33	—
	ii	9·30	1·00	0·80	28·11	—
	iii	9·30	1·00	0·80	30·20	26·28
	Mean	9·32	1·00	0·80	28·55	—
E	iv	9·18	1·055	0·815	25·21	—
	v	9·12	1·055	0·815	25·64	—
	vi	9·13	1·03	0·80	26·02	32·41
	Mean	9·14	1·05	0·81	25·62	—

The specific gravities found for two of the magnets were approximately equal, the mean being 7·67.

From this we conclude that in both groups of magnets the wall thickness is less at the extreme ends, where an internal screw is cut, than elsewhere; the mean internal diameter deduced from the data being 0·70 cm. for the magnets of group C, and 0·77 cm. for those of group E. The actual volumes of steel in the two cases were respectively 3·72 and 3·34 c.c.

There has been but little variability in the length or external diameter of the magnets of any one group; the volume however, if we may judge by Table III, is somewhat more variable. We should have more exact information on the point if K meant the moment of the magnet alone, but as matters stand, our conclusions are exposed to some uncertainty. The appendages, as we see from Table III, weigh about as much as the magnets themselves, their main mass lies however nearer to the axis of rotation. In fact, according to the measurements

made on the special magnets the appendages contribute only from 25 to 30 per cent. of the value of K.

Again, the size of the appendages is determined mainly by that of the auxiliary brass bar, which according to the Observatory records is nearly constant for magnets of the same make. It is thus improbable that variations in the appendages exercise a large influence on the values of π^2K in the several groups.

§ 8. Taking all the data into consideration, I conclude from Table I, that the magnets of group A are on the whole distinctly the largest, and those of group F very distinctly the smallest; that the other four groups are very fairly similar in this respect, though the average magnet of group E is almost certainly a trifle smaller than that of groups B and C. The variability in size was really one of my principal reasons for grouping the magnets. The values of m and μ should increase, *ceteris paribus*, with the size of the magnets, and an analysis which overlooked the difference between groups A and F might well prove misleading. So far as individual groups are concerned, I do not think that the neglect of possible variations in the size of the magnets is likely to prove serious. My reasons for this opinion are partly based on phenomena discussed later in §§ 15 and 16.

§ 9. Before proceeding further, a second source of uncertainty must be noticed. The moment of the residual magnetism in a magnet is not determined solely by the magnetic quality of the steel, it depends on the conditions under which magnetisation took place, on the time elapsed since that event, and on the usage to which the magnet has been exposed.

In the present case I have strong reasons to think that the very large majority at least of the magnets were magnetised under a nearly uniform set of conditions. Until quite lately it was the invariable practice to magnetise all the magnets at the Observatory itself, and the same coil with similar battery power has been in use for the last forty years. The magnet is not placed inside the coil, but is stroked in a uniform way on the end of a very massive slightly projecting iron core. The capacity of this core to hold out a heavy pole piece has been regarded as a criterion of the battery being in proper order; and as apparent "saturation" is reached in an ordinary collimator magnet with the battery below standard condition, the test, though a rough one, appears fairly satisfactory.

There is a variety of indirect evidence as to the uniformity of the conditions. For instance, the dates of magnetisation of the ten magnets of largest m in group C, arranged in descending order of magnitude, were as follows:—1884, '66, '79, '73, '95, '84, '83, '94, '75, '89, '68.

As regards possible loss of magnetic moment, the magnets, with one or two probable exceptions, had all been magnetised but a short time before m and the various "constants" were determined.

§ 10. *Magnetic Moment.*—Taking into account the variability in size which we have decided to exist, it would appear from Table I that the capacity for assuming permanent magnetism has been nearly the same in the four groups A, C, D, and E; but the magnets of group B have been distinctly less receptive than the others. It is difficult to say exactly how the magnets of group F stand. They are clearly much below the average size, and their mean m is at least as large compared to the means in the other groups as one would expect from a comparison of the mean moments of inertia.

Table II shows the variability of m to have been much greater in each group than that of K.

The great variability in m is noteworthy, because when m is small, corrections for torsion, temporary induction, &c., increase in relative importance.

It has been the practice of late years to reject magnets showing exceptionally small magnetic moment, but the data from such rejected magnets are not included in Tables I and II.

§ 11. *Temperature Coefficients.*—Out of the whole 130 magnets there was not a single case in which the arithmetic mean of the values found experimentally for q' was negative. Thus within the experimental limits, say 0° to 36° C., the rate of change of magnetic moment with temperature invariably increased with the temperature.

The mean value for q in groups C, E, and F, is practically identical, and is very distinctly larger than the mean values for groups B and D. Group B is specially remarkable for the smallness of the temperature coefficients; in fact, the largest value of q found in a magnet of this group is very little greater than the mean value in groups A, C, E, and F.

The mean value of q' is fairly similar for the different groups, but groups A and B have somewhat smaller values than the others.

The variability of both q and q' is, as we see from Table II, very considerable in all the groups except E, where q varies but little. The term $q't^2$ is in general small compared to the term qt, and consequently the probable error in the determination of q' is large. The variability of q' may for this reason be somewhat exaggerated in Table II.

§ 12. *Induction Coefficient.*—As explained above, μ is the product of the volume of the magnet into its permeability, as determined by reversing a field of about 0·44 C.G.S. unit, the magnet being possessed of a large permanent magnetic moment.

Table I shows that the size of the mean μ in the first five groups has little relation to the size of the mean m. This is significant, because μ and m should vary in the same way with the volume of the magnet.

At the same time, the exceptionally small volume undoubtedly possessed by the magnets of group F is almost certainly the cause of the small μ found in that group; and we shall probably be correct in

concluding that the larger mean μ possessed by group A as compared to groups B, C, or D, is due to the possession of a larger volume by the first-mentioned group. The pre-eminence of the mean μ of group E is, however, explicable only by supposing that the magnets of that group possess a considerably larger permeability than the others; and this larger permeability, as we see from Table II, is a property not of one or two magnets, but of the whole group.

On the whole, as Table II shows, μ is much less variable in the several groups than q or q'.

The greatest recorded value of μ, viz., 11·89, occurs in group D. I feel, however, some doubts respecting this and a second large value, 9·72, found for a second magnet of the same group. The two magnets for which these values are recorded were tested in 1864; and one of them, when retested many years afterwards, had a μ less than 7. If we excluded these two magnets, we should find for group D a mean μ much the same as that of group C.

§ 13. *Coefficient* P.—In about half the cases P was determined from only one observation, and under such circumstances the probable error is considerable. Still I do not think that the mean values given in Table I can be much in error. It will be noticed that no magnet of group B gave a positive P, and that no magnet of groups E or F gave a negative P. In groups A, C, and D, positive and negative values occur in fairly similar proportions; but investigation showed a great predominance of negative values amongst the older magnets of group C, while in group D no positive value appeared in the six oldest magnets. As pointed out in § 5, P depends on the deflected or mirror magnet as well as on the collimator magnet, and I am inclined to ascribe the interesting difference between older and newer unifilars mainly to change in the pattern of the mirror magnets (see § 46 later).

As appears from either Table I or Table II, P when negative is usually numerically smaller than when positive. When an observer employs a unifilar strange to him the probable error in P is doubtless larger than when the instrument is one to which he is thoroughly accustomed, and it would not be unreasonable to attribute to this cause some of the large values recorded in Table II. It happens, however, that three of the largest values of P in group C were each based on three independent observations. The results of the individual experiments were as follows, the numbering being purely arbitrary :—

Unifilar.	i.	ii.	iii.
Values of $10^5 \times \mathrm{P}/r^2$ at 30 cm.	+2415 +2870 +2303	−1439 −1917 −1107	+1031 + 872 + 904

The probable errors in the mean values found in these three cases are certainly sensible, but still they form only a comparatively small percentage of the value of P. Certain drawbacks attending large values in P are dealt with later.

§ 14. A general survey of Tables I and II shows merits and demerits in most of the groups of magnets. Thus in group B the temperature coefficients are exceptionally small, but on the other hand the magnets are somewhat weak. Again, in group E the magnets are exceptionally uniform in quality, but they possess exceptionally large permeability for temporary magnetism.

Relationships between Magnetic Constants.

§ 15. In attempting to determine the existence or non-existence of relationships between the magnitudes of the several magnetic constants, one naturally turns first to the large group C. To determine whether the size of the permanent magnetic moment influenced the other quantities, I arranged the eighty-two magnets of the group in sub-groups as below, and found the mean values of the several constants for each.

Table IV.—Analysis of Magnets of Group C according to Magnetic Moment.

Value of m.	Number in sub-group.	Mean m.	Mean π^2K.	Mean 10^6q.	Mean $10^8q'$.	Mean μ.	$\frac{\text{Mean } m}{\text{Mean } \mu}$.	Mean $10^5 \times \mathrm{P}/r^2$. When +.	When −.
>1000	11	1044	2812	346	140	5·96	175	1399	481
1000 to 950	10	973	2745	313	143	5·53	176	884	304
950 ,, 900	11	921	2702	331	135	5·72	161	849	188
900 ,, 875	9	888	2759	359	149	5·61	158	628	286
875 ,, 850	12	862	2809	352	144	5·65	153	859	680
850 ,, 800	10	827	2720	300	147	5·42	152	613	350
800 ,, 750	10	779	2791	365	156	5·57	140	757	257
<750	9	653	2684	433	137	5·58	117	472	298

It is only natural to expect a sensible departure between the means of any single property in a sub-group of ten magnets and in a whole group of eighty-two magnets, supposing the sub-group selected by pure chance. Bearing this in mind, we must, I think, conclude that no clear relationship exists between the value of m and that of K, q, q', or μ. The absence of apparent connection between m and π^2K seems strong evidence of the comparatively small variability in the actual size of the magnets. The fact that μ is nearly, if not quite, independent of m is important, because it is expedient *ceteris paribus* that m/μ should be as large as possible. If q, q', or μ had shown an appreciable tendency to vary with m we should have been led to suspect, as *à priori* probable, a tendency to alteration in these constants as the magnet grew weaker with age. The absence of any apparent connection of the kind is not, however, proof positive that no such tendency exists (see § 37 later).

§ 16. I next arranged the eighty-two magnets of group C in sub-groups, according to the value of μ, with the following results:—

Table V.—Analysis of Magnets of Group C according to Size of Induction Coefficient.

Value of μ.	Number in sub-group.	Mean μ.	Mean m.	$\frac{\text{Mean } m}{\text{Mean } \mu}$.	Mean π^2K.	Mean 10^6q.	Meon $10^8q'$.	Mean $10^5 \times P/r^2$. When +.	When −.
>6	26	6·32	883	140	2762	433	160	800	359
6 to 5·5	24	5·71	898	157	2704	330	133	994	245
5·5 ,, 5	19	5·25	854	163	2796	294	132	754	241
5 ,, 4	13	4·69	837	179	2774	296	148	787	931

There is here no trace of a connection between μ and K, which supports the conclusion drawn from the absence of apparent connection between m and K. There is probably a slight tendency when μ is very decidedly below the mean for m to be slightly low; but the tendency in m/μ to increase as μ diminishes is conspicuous.

The large value for P, when negative, when μ lies between 5 and 4 has no real significance, the mean being based on only two magnets. The one clear and important relationship brought out by Table V is between μ and q. Large values of these two constants unquestionably have a tendency to occur together in the magnets of group C. Out of the twenty-six magnets whose μ exceeded 6, no less than eighteen had a q above the mean; while of the thirteen magnets whose μ fell short of 5, only two had a q above the mean. As the result seems important, I submit the following analysis, showing the distribution of the different values of μ in group C:—

Table VI.

μ.	10^6q = 150	200	250	300	350	400	500	600	700	800
All values....	....	4	11	17	19	13	7	8	1	2
From 7 to 6..	....	0	2	3	3	6	4	5	1	2
,, 6 ,, 5·5	....	2	3	3	7	5	2	2	0	0
,, 5·5 ,, 5..	....	2	3	6	5	2	0	1	0	0
,, 5 ,, 4..	....	0	3	5	4	0	1	0	0	0

The table is to be read thus: Of the eighty-two magnets, four had a value of 10^6q between 150 and 200, and of these, two had a μ lying between 6 and 5·5, while two had a μ lying between 5·5 and 5.

Table VI certainly supports the conclusion drawn from Table V.

As a further check on this conclusion, I arranged the eighty-two magnets in two sub-groups, according as q was above or below the mean, with the following result:—

Table VII.

q.	Number in sub-group.	Mean 10^6q.	Mean $10^8q'$.	Mean μ.	Mean m.	Mean $10^5 \times P/r^2$. When +.	When −.
Above mean ...	33	457	160	5·94	873	854	303
Below mean ...	49	276	133	5·43	873	836	387

Table VII, like the two previous tables, points to some connection between large values of μ and q. Out of the first twenty magnets, the arrangement being in descending order of q, only two had a value of μ below the mean for group C.

The association of large values of μ and q in group C is not, however, without some conspicuous exceptions; for instance, the magnet coming eighteenth in the list just referred to had a μ of only 4·71.

§ 17. There are several other interesting features in Table VII; *e.g.*, only about two-fifths of the magnets had a q above the mean. Again the fact, partly accidental of course, that the two sub-groups should have precisely the same mean m is strong evidence that in the magnets of group C the size of the principal temperature coefficient is independent of the capacity of the steel to retain a large magnetic moment.

The mean q' of the first sub-group of Table VII is distinctly larger than that of the second sub-group, but the evidence of a tendency in large values of q and q' to go together is not wholly conclusive. Thus out of the thirty-three magnets whose q exceeded the mean, fourteen had a q' below the mean; and two of these fourteen had the largest q's of the group. The force of such notable exceptions is weakened, however, by the consideration that an experiment which makes q slightly too big is more likely than not to make q' considerably too small.

§ 18. Groups B and E are the only ones beside C which are sufficiently numerous and homogeneous to merit analysis. In the two following tables I give the results obtained by subdividing each of these groups into two numerically equal sub-groups, according to the size, first of m, second of μ.

Table VIII.—Mean Values of Constants in Sub-groups.

Group.	Sub-group in order of m.	m.	π^2K.	10^6q.	$10^8q'$.	μ.	$\frac{\text{Mean } m.}{\text{Mean } \mu.}$	$10^5 \times P/r^2$.
B	First 6...	759	2850	273	135	6·10	124	−475
	Second 6.	584	2685	241	102	5·96	98	−395
E	First 5...	883	2529	336	162	7·56	117	+737
	Second 5.	791	2511	362	150	7·56	105	+572

Table IX.—Mean Values of Constants in Sub-groups.

Group.	Sub-group in order of μ.	μ.	m.	$\frac{\text{Mean } m.}{\text{Mean } \mu.}$	π^2K.	$10^6 q$.	$10^8 q'$.	$10^5 \times P/r^2$.
B	First 6...	6·48	674	104	2929	256	141	−487
	Second 6 .	5·58	669	120	2606	257	96	−385
E	First 5...	7·70	860	112	2575	345	141	+909
	Second 5 .	7·42	815	110	2465	353	171	+401

Values of P were wanting for three of the twelve magnets of group B.

In one respect Tables VIII and IX unquestionably agree with the results established for group C; they show a distinct tendency in m/μ to be large when m is large. There is, however, in Table IX no association of large values of μ and q. In the case of group B there is an apparent association of large values of both m and μ with large values of K, suggesting that the size of the magnets has exerted a slight but sensible influence. In the case of group E the differences between the various magnets are so extremely small that we could hardly hope to detect any relationship that was not very intimate and potent.

The previous part of the paper has been mainly historical and descriptive; it remains to consider the subject from a more critical standpoint.

Probable Errors in Determination of Horizontal Force due to Errors in Values of "Constants."

§ 19. Let us first assume that the methods of determining the "constants" and the formulæ employed in calculating the horizontal force are alike above suspicion, and investigate on this hypothesis the uncertainties introduced by the probable errors in the values found for the several "constants." To do this, we mnst consider the formulæ. The notation not already explained is as follows:—

X = horizontal component of magnetic force;
T_1 = semi-period of vibration, corrections having been applied, if necessary, for rate of chronometer and for finite arc of vibration;
Θ = torsion couple, during vibration experiment, when torsion angle unity;
u = deflection angle, during deflection experiment, when r distance apart of magnets' centres;
t = temperature during vibration experiment;
t' = " " deflection " .

For clearness, I shall in what follows suppose r to be 30 cm., that being the smaller of the two distances now adopted generally.

X is deduced by combining the two formulæ—*

$$mX = \frac{\pi^2 K}{T_1^2}\left\{1+\frac{\Theta}{mX}-qt-q't^2+\frac{2\mu \operatorname{cosec} u}{r^3}\right\}^{-1} \quad \ldots\ldots \quad (2),$$

$$m/X = \left(1-\frac{P}{r^2}\right)\left\{1+\frac{2\mu}{r^3}+qt'+q't'^2\right\}\tfrac{1}{2}r^3 \sin u \quad \ldots\ldots\ldots \quad (3)$$

whence we have—

$$X^2 = \frac{\pi^2 K \,.\, 2r^{-3} \operatorname{cosec} u \,(1-P/r^2)^{-1} T_1^{-2}}{\left(1+\frac{\Theta}{mX}-qt-q't^2+\frac{2\mu}{r^3}\operatorname{cosec} u\right)\left(1+\frac{2\mu}{r^3}+qt'+q't'^2\right)} \quad \ldots \quad (4).$$

Suppose δK, $\delta\mu$, &c., to represent the errors in the values ascribed to K, μ, &c. Terms in Θ, μ, q, q' in the denominator of the expression for X^2 are always small compared to unity, and may for our present purpose be neglected in the coefficients of $\delta\mu$, δq, &c. In this way we easily find for the consequent error δX in X :—

$$\frac{\delta X}{X} = \tfrac{1}{2}\frac{\delta K}{K}+\tfrac{1}{2}\frac{\delta P}{r^2}-\delta\mu\frac{(1+\operatorname{cosec} u)}{r^3}+\frac{t-t'}{2}\{\delta q+(t+t')\,\delta q'\}-\tfrac{1}{2}\delta\left(\frac{\Theta}{mX}\right) \quad \ldots\ldots\ldots \quad (5).$$

From (5) we see noteworthy differences in the consequences of errors in the different "constants."

In the case of K it is not the absolute size of the error that counts, but the ratio it bears to the size of K; while in the case of P, μ, and the temperature coefficients it is the absolute size of the error that counts. In all cases δX increases with X, so that the *absolute* effect of a given error in any one of the "constants" is greater where the horizontal force is large than where it is small.

§ 20. In estimating the probable errors in the several constants, I have confined my attention to the cases in which the accepted results were based on two, and only two, experiments.

If $2\delta y$ be the difference between the values given by two experiments for a certain quantity, the probable error in the arithmetic mean, y, of the two determinations is

$$\delta y'' \equiv \delta y \times 0{\cdot}6745.$$

In many instances the value of a "constant" was based on only one experiment. In such cases we may reasonably assume that the single experiment was, on the average, neither better nor worse than the

* *Cf.* Stewart and Gee's 'Elementary Practical Physics,' vol. 2, pp. 298, 307, &c., allowing for difference of notation.

average experiment which formed one of the couple usually made. This would give for the probable error in the value of a "constant" based on one experiment only, the value

$$\delta y' = (2)^{\frac{1}{2}} \overline{\delta y''},$$

where $\overline{\delta y''}$ is the arithmetic mean of the probable errors $\delta y''$ found in the cases where two experiments were made.

In the few cases where constants were based on three independent experiments, one could, of course, have calculated the probable errors $\delta y'''$, and found their mean. The number of such cases appeared, however, too small to give a satisfactory mean value.

§ 21. Taking first the moment of inertia, I examined seventy-one cases in which two independent determinations had been made. Employing $2\delta K$ to represent the difference between the two observed values, whose arithmetic mean is K, I found

$$\text{Mean } \delta K/K = 0{\cdot}00041.$$

The corresponding mean probable error is given by

$$\delta \overline{K}''/\overline{K} = 0{\cdot}000277,$$

and answering to this the probable error in X is

$$\delta X = 0{\cdot}000138X.$$

Ascribing to X the value 0·18 C.G.S. unit—which is not far from the present mean value in Great Britain—we should have

$$\delta X = 0{\cdot}000025, \text{ approx.}$$

Taking the same value 0·18 for X, I also determined the law of incidence of the probable error in the seventy-one cases examined.

The results were as follows :—

Table X.

(Prob. error) × 10^5 between	0 and 0·5	0·5 and 1	1 and 2	2 and 3	3 and 4	4 and 5	greater than 5
Number of magnets	18	8	13	14	4	6	8

As measurements of horizontal force are usually taken to 1×10^{-5} C.G.S. unit, we see that error in the moment of inertia may be expected to affect the last significant figure, in these latitudes, in fifty-three cases out of seventy-one, or practically in two cases out of three. In one-ninth of the total number of cases the probable error in X shown by Table X reached or exceeded five units in the last significant figure.

Also it must be remembered that in equatorial regions X may be double the value assumed in Table X, and that δX in this case varies directly as X.

§ 22. The number of instances where two observations had been made of μ were fewer; I examined forty-one of these in all.

Representing by $\delta\bar{\mu}$ and $\delta\bar{\mu}''$ the mean of the semi-differences between the two observed values and the mean probable error respectively, I found

$$\delta\bar{\mu} = 0{\cdot}198, \qquad \delta\bar{\mu}'' = 0{\cdot}134.$$

The corresponding probable error in X, treating r as 30 cm., is, irrespective of sign,

$$\begin{aligned}\delta X &= X(1+\operatorname{cosec} u)(30)^{-3}\times 0{\cdot}134,\\ &= 5\times 10^{-6}X(1+\operatorname{cosec} u), \text{ approx.}\end{aligned}$$

This is troublesome to deal with; because cosec u depends both on X and on the magnetic moment. As a first approximation we have in fact

$$\operatorname{cosec} u = X\times 30^3/2m.$$

To get an idea of the probable error arising from error in μ, suppose $X = 0{\cdot}18$, as at Kew, and $m = 840$, as in the average new collimator magnet. This gives

$$\operatorname{cosec} u = 3, \text{ approx.}$$

$$\delta X = 0{\cdot}000004, \text{ approx.}$$

Thus when there are two observations of μ the probable error will in the average case fail to affect the last significant figure, supposing X measured as usual to 1×10^{-5} C.G.S.

Of course in a good many individual cases the probable error in μ, determined from two observations, was sufficient to affect the last significant figure at Kew. More often than not μ has been derived from a single experiment, and in the majority of such cases we should conclude that the probable error was large enough to affect the last significant figure in X, measured at Kew. Owing to the occurrence of a term

$$-r^{-3}X\operatorname{cosec} u \equiv -X^2/2m$$

in the expression for $\delta X/\delta\mu$, we see that where X is large, and m is small, error in μ may be very much more serious than in the case we have selected for numerical treatment.

§ 23. The influence of errors in q and q' on X is difficult to present clearly, as it depends both on the difference and the mean of the temperatures t and t' existing during the vibration and deflection experiments. If t and t' are equal, it does not matter—the funda-

mental formulæ being granted—how largely q and q' are in error: but this is an exceptional occurrence, especially in field observations. To consider all possible mean temperatures seemed unnecessary, and I thus confined my attention to the three cases

$$(t+t')/2 = 0, \qquad = 15^\circ \text{ C.}, \qquad = 30^\circ \text{ C.};$$

corresponding to which,

$$\delta q + (t+t')\,\delta q' = \delta q, \qquad = \delta(q+30q'), \qquad = \delta(q+60q').$$

In other words, I found the difference between the two observed values of the three quantities q, $q+30q'$ and $q+60q'$, treated independently, and the corresponding three independent probable errors.

The results, derived from seventy magnets, were as follows:—

Quantity.	$10^6 q$.	$10^6(q+30q')$.	$10^6(q+60q')$.
Mean semi-difference	11·5	7·8	14·1
Mean probable error	7·8	5·3	9·5

Corresponding to this, we have for the mean probable errors in X

Mean temperature	0° C.	15° C.	30° C.
δX.........	$3·9\,(t-t')\,10^{-6}$X	$2·6\,(t-t')\,10^{-6}$X	$4·7\,(t-t')\,10^{-6}$X.

The probable error is conspicuously less near the middle part of the temperature range covered by the actual experiment than near either limit of this range; and this is only what we should anticipate. When the mean temperature during a horizontal force observation, at Kew, is 15° C., it would in the average unifilar require a difference of fully 10°C between the mean temperatures during the vibration and deflection experiments to make the probable error in q and q' affect the fifth significant figure in X. So large a temperature difference as this need hardly ever be feared in a fixed observatory.

The result is so far comforting, but does not justify the conclusion that error in the temperature coefficients is a wholly improbable cause of error in X. In some individual cases the probable errors found for q and q' were five or six times larger than the mean. Again, in a considerable number of instances, q and q' have been derived from only one experiment. Finally it should be noticed that the probable error δX increases with X, and that on the whole X is largest in equatorial regions where the temperature is high, and consequently errors of given magnitude in q and q' most effective.

§ 24. The two terms $\frac{1}{2}(\delta\text{P}/r^2)$ and $-\frac{1}{2}\delta(\Theta/m\text{X})$ in (5) were included with the object of showing how errors in the values assigned to P or to the torsion affect X. I have, however, no satisfactory data as to the size of the probable errors in P or the torsion coefficient under normal conditions. The torsion coefficient varies from thread to thread, and also with the dampness of the air. It is in fact treated as variable, and is usually

determined specially in each observation of X. The accuracy of the determination thus depends more on the observer than on the instrument.

P has never been regarded at Kew as a constant to be determined once for all, but observers are recommended to determine it for themselves, employing, when possible, a series of observations made about the same time and in one neighbourhood. In other words, P is treated as a quantity which probably alters slowly with the time, and which may vary with X. In the Kew unifilar itself the mean value of P calculated from a year's observations varies slightly but somewhat irregularly from year to year.

The degree of variation is best seen by consulting the following table:—

Table XI.

Year.	$-10^5 P/r^2$ at 30 cm.	Year.	$-10^5 \times P/r^2$ at 30 cm.	Year.	$-10^5 \times P/r^2$ at 30 cm.
1860	198	1886	153	1892 } 1893 }	67
1875	185	1887	192	1894	140
1879	112	1888	174	1896	131
1882 } 1883 }	133	1889	211	1897	111
1884	172	1890	177	1898	143
1885	95	1891	121		

Each one of these results is based on a large number of observations, but I should hesitate to say that observational error plays no part in the apparent fluctuations. There are of course frequently minor magnetic disturbances during horizontal force observations, and two or three outstanding values of P sensibly affect a mean though derived from forty observations. Though the annual mean has invariably made P negative, positive values from individual observations are by no means uncommon.

If we consider that the data in the above table are based each on numerous experiments, taken at a fixed station under the best conditions, we must, I think allow that uncertainty in the P correction is a very probable source of trouble in survey work.

Criticism of Formulæ from Mathematical Standpoint.

§ 25. If θ be the angle made by the axis of the collimator magnet with the magnetic meridian at time τ, the equation of motion is

$$K'\frac{d^2\theta}{d\tau^2}+\Theta\theta+\{mX(1-qt-q't^2)+\mu X \,.\, X\}\sin\theta = 0 \quad \ldots \quad (6),$$

where K′ is the moment of inertia of the magnet and appendages, and μ the induction coefficient at temperature t.

If θ be kept small enough this gives an isochronous vibration whose half period T_1 is given by

$$T_1{}^2 = (\pi^2 K'/mX) \div \left(1 + \frac{\Theta}{mX} - qt - q't^2 + \frac{\mu X}{m}\right),$$

whence $$mX = (\pi^2 K'/T_1{}^2) \div \left(1 + \frac{\Theta}{mX} - qt - q't^2 + \frac{\mu X}{m}\right) \quad \ldots\ldots\ldots \quad (7).$$

Unless θ be kept very small, a correction becomes necessary for "finite arc" of vibration; and we then encounter the difficulty that the torsion couple is $\Theta\theta$ and not $\Theta \sin\theta$.

This is rarely, however, of practical importance, except at places where X is specially small, supposing one avoids coarse suspension fibres. At Kew Θ/mX very seldom reaches 0·001, and θ need never exceed 2°; and under such conditions $\Theta \sin\theta$ may be freely written for $\Theta\theta$. The correction for "finite arc" then presents no peculiarity.

A second criticism that may be passed is that (6) makes no allowance for air resistance; in the absence of experimental data I have nothing to say on this point.

§ 26. In the deflection experiment the deflecting and deflected magnets are at right angles, the latter making an angle u with the magnetic meridian.

Supposing t' the temperature, μ' the induction coefficient of the collimator magnet during this experiment, and X′ the horizontal force, we have—

$$X'/m = 2r^{-3} \operatorname{cosec} u \{1 - qt' - q't'^2 - (\mu' X'/m) \sin u\} (1 + Pr^{-2}) \ldots (8),$$

assuming the term Qr^{-4} negligible.

Here r is the actual distance at temperature t' between the centres of the two magnets.

Unless a large magnetic storm is in progress—in which case an absolute observation of horizontal force is worthless—we may regard X and X′ as equal in the small terms of (7) and (8), and so may write these equations as

$$mX = (\pi^2 K'/T_1{}^2) \div \{1 + (\Theta/mX) - qt - q't^2 + 2\mu r^{-3} \operatorname{cosec} u\},$$

$$X'/m = 2r^{-3} \operatorname{cosec} u \, (1 + Pr^{-2}) (1 - qt' - q't'^2 - 2\mu' r^{-3}).$$

Thence, eliminating m, we have

$$XX' = \frac{2\pi^2 K' \operatorname{cosec} u}{r^3 T_1{}^2} \, \frac{(1 + Pr^{-2})(1 - qt' - q't'^2 - 2\mu' r^{-3})}{1 + (\Theta/mX) - qt - q't^2 + 2\mu r^{-3} \operatorname{cosec} u} \ldots (9).$$

This differs from (4) in several respects; but some of these possess little real significance.

Thus we have XX' in (9) as against X^2 in (4); but this only means that the X appearing in (4) is in reality a mean between the values possessed by the horizontal force during the vibration and deflection experiments.

In actually comparing the result of an absolute horizontal force observation with the magnetogram one measures the curve ordinates at the mean times of the two experiments—times separated usually by thirty or forty minutes—and takes the mean of the two ordinates as corresponding to the X deduced from (4).

Again, the K appearing in (4) is really taken from a table which allows for variations of temperature, and the same is true of r.

§ 27. The first difference of real significance is that (9) contains $1+\mathrm{P}r^{-2}$, while (4), on the other hand, has $(1-\mathrm{P}r^{-2})^{-1}$. Supposing X measured as usual to five significant figures, this becomes objectionable when the value of X is affected by so much as 5×10^{-6} C.G.S. unit.

The limiting value of $\mathrm{P}r^{-2}$ for which the substitution is justifiable is given by

$$X(1-\mathrm{P}^2r^{-4})^{\frac{1}{2}} = X - 5\times 10^{-6},$$

or, approximately, $$\mathrm{P}r^{-2} = 10^{-3}(10/X)^{\frac{1}{2}}.$$

This gives—

$$\text{For } X = 0{\cdot}18,\ \mathrm{P}r^{-2} = 0{\cdot}0074, \text{ approx.,}$$
$$X = 0{\cdot}36,\ \mathrm{P}r^{-2} = 0{\cdot}0053 \quad ,,$$

The mean value found for $\mathrm{P}r^{-2}$, when P is positive, in Table I is just in excess of 0·0074, and values in excess of 0·0053 are very common. Thus the employment of $(1-\mathrm{P}r^{-2})^{-1}$ in (4) is, to say the least of it, frequently unjustifiable.

As regards the possible size to which the error in question might attain, we should have in the case of the largest P given in Table II

$$\text{for } X = 0{\cdot}18, \text{ error } 0{\cdot}00006,$$
$$X = 0{\cdot}36, \quad ,, \quad 0{\cdot}00011.$$

§ 28. The second difference of note between (4) and (9) is that $(1+qt'+q't'^2+2\mu r^{-3})^{-1}$ occurs in the former as against

$$1-qt'-q't'^2-2\mu' r^{-3}$$

in the latter.

Overlooking for the present a possible difference between μ and μ', we see that objection arises when

$$(qt'+q't'^2+2\mu r^{-3})^2$$

ceases to be negligible.

So far as the term in μ is concerned there is no cause for apprehension, as 0·0006 would be an exceptionally large value for $2\mu r^{-3}$.

The case of the temperature terms is less satisfactory. Taking, for example, the mean values of Table I, viz.,

$$q = 335 \times 10^{-6}, \qquad q' = 14 \times 10^{-7},$$

we have

$$\begin{array}{rccc} t' = & 10^\circ & 20^\circ & 30^\circ \\ qt' + q't'^2 = & 349 \times 10^{-5} & 726 \times 10^{-5} & 1131 \times 10^{-5} \end{array}$$

Following exactly the same procedure as in the case of $\mathrm{P}r^{-2}$, we thence conclude that the treatment applied to the temperature terms will on the average begin to introduce error, supposing $\mathrm{X} = 0{\cdot}18$, when t' reaches 20° C.

When a magnet with large temperature coefficients is used in hot weather at a place where X is large, the error due to the treatment of the temperature terms becomes very sensible. Suppose, for instance,

$$q = 6 \times 10^{-4}, \qquad q' = 22 \times 10^{-7},$$

values exceeded in one actual case, then we have

$$\text{for } t' = 35^\circ,$$

$$qt' + q't'^2 = 0{\cdot}0237.$$

This gives an error of approximately 0·0001 C.G.S. unit when $\mathrm{X} = 0{\cdot}36$.

*Criticism from Physical Standpoint.**

§ 29. Under this heading I shall discuss certain grounds of uncertainty which may affect the accuracy of the calculated values of the horizontal force. The considerations are mainly, but not exclusively, physical.

An objection to both the fundamental formulæ (7) and (8) is that they represent all the quantities involved as constant, whereas in general magnetic force and declination, temperature, and moment of magnet, are constantly altering.

In reality, however, the t appearing in (7) is the mean of two readings of a thermometer adjacent to the collimator magnet, taken, the one immediately before, the other immediately after, the vibration experiment; while the t' appearing in (8) is the mean of eight thermometer readings taken nearly simultaneously with the individual observations of the deflection angles. There is thus very fair provision for changes of temperature so long as the temperature changes slowly and continuously in one direction, provided the thermometers really record the temperature of the magnet. In a fixed observatory these conditions are probably as a rule fairly secured if the magnet and thermometer be freely exposed to the air for five or ten minutes before the experiment commences.

* November 22.—Compare a paper by Wild (which I had overlooked), 'Terrestrial Magnetism,' vol. 2, 1897, pp. 85—104.

§ 30. Variations of horizontal force and declination—which likewise affects both vibration and deflection readings—are more serious. If small and gradual, they are of course likely to be largely eliminated, especially in the deflection experiment, where the order of the operations is specially devised with a view to this end. If, however, they are large and sudden, probably the only really satisfactory course is to reject the observation.

It might be supposed that at a fixed observatory corrections could always be applied by referring to the magnetograph curves. This, however, is not very feasible in practice. The time scale of the curves is so contracted that the time to which a particular ordinate refers is usually uncertain to the extent of at least half a minute. Again, the position at any instant of a magnet during rapid changes of field depends to an extent difficult to determine on the inertia of the magnet; and the magnets used in magnetographs and in absolute instruments differ greatly from one another both in inertia and in method of suspension. The vibration of a magnet under sudden irregular variations of horizontal force and declination is in all probability a very complicated problem.

§ 31. The possibility of obtaining satisfactory horizontal force measurements depends largely on avoiding times of serious magnetic disturbance.

The shorter the time occupied by the observations, the better the chance of accomplishing this. Supposing a uniform procedure adopted, the time occupied by a vibration experiment is independent of the observer; it is shorter the stronger the magnet and the greater the horizontal force. It is thus of especial importance that collimator magnets intended for work in arctic regions, where the force is low and disturbances large and numerous, should be of high magnetic moment.

In the deflection experiment a great deal depends on the skill of the observer, and on the make of the unifilar; one observer may do in twenty minutes what occupies another for fifty. Extreme conscientiousness may be a positive demerit, owing to the excessive time spent in adjustments and readings.

A variety of other criticisms will be considered which apply to some one physical quantity, and are classified accordingly.

§ 32. *Moment of inertia.*—In default of any simple means of directly measuring the variation with temperature of the moment of inertia, this variation is calculated by assuming for the coefficients of linear expansion:—

18×10^{-6} in the auxiliary brass bar,

12×10^{-6} ,, ,, collimator magnet.

There is of course the uncertainty that the assumed mean values may

not apply very exactly to the particular samples of brass and steel employed in any individual case; and there is the further criticism that the coefficient 12×10^{-6} is applied to a composite moment of inertia K, of which 25 to 30 per cent. is contributed by the magnet's appendages, which are mainly brass. The expansion coefficients are employed first in deducing the value of K at 0° C. from the value found at some higher temperature, and then in calculating from the value at 0° C., thus found, a table giving K at all temperatures likely to occur in actual use. Any error that may arise in this way will be larger the more remote the temperature from that existing during the special inertia experiments. Its probable magnitude cannot, however, be directly arrived at in the absence of measurements of the expansion of specimen magnets, their appendages and auxiliary bars.

A second source of uncertainty is the fact that one is obliged to assume uniform density in the auxiliary bar.

§ 33. *Torsion of suspending thread.*—This is required (1) in reducing the declination experiment, (2) in the vibration experiment determining the horizontal force.

For a considerable time prior to a declination experiment at Kew, the suspending silk fibre is stretched by hanging from it a non-magnetic plummet similar in weight to the magnet and its appendages. As the magnetic meridian is very approximately known, it is easy to arrange that when the plummet is replaced by the declination magnet there shall be very little twisting of the silk; we may thus, with at least reasonable probability, regard the thread at the beginning of the experiment as practically free from torsion. After the experiment is concluded, the magnet is again replaced by the plummet, and after some hours its position is noted. In this way we can tell approximately the angle θ' through which the lower end of the thread has turned since the last declination reading was taken. It is assumed that θ' represents the amount of torsion in the silk when the experiment ended, and that this torsion was introduced gradually in the course of the experiment. As the magnet has to be variously manipulated, there is no antecedent mprobability in the hypothesis; but it will, I think, be recognised that the taking $\theta'/2$ as the mean torsion angle during the experiment is an emendation whose success is likely to be variable.

To allow for $\theta'/2$ of torsion, one introduces a given twist into the thread and notes the consequent change of azimuth in the suspended magnet. The procedure adopted at Kew, is to turn the suspension head carrying the thread through 180° first in one direction then in another, noting the corresponding equilibrium positions of the magnet.

§ 34. The occasion for a torsional correction in the vibration experiment is the existence of the couple $\Theta\theta$ in (6), and consequently of the term $\Theta/m\mathrm{X}$ in (7). The value of $\Theta/m\mathrm{X}$ is deduced from the observed changes in the azimuth of the collimator magnet when the suspension

head is turned through $\pm 180°$. There are several possible criticisms. In the actual vibration experiment, the thread is twisted at most only a degree or two, and it is open to doubt whether a value found for Θ from twists through $\pm$ 180° is strictly applicable, even supposing the conditions otherwise identical.

There is usually more than one silk fibre in the suspension, and this increases the probability that the value found statically for Θ may not apply exactly to a vibration experiment.

Another criticism is that experiment really gives $\Theta/m'X'$, where m' and X' are the values of the magnetic moment and of the force during the torsion experiment, instead of Θ/mX, where m is the moment at 0° C. and X the force during the vibration experiment.

To judge of the effect of this, we require the relation between the error in Θ/mX and the consequent error in X. For this we have from (5)

$$\delta X = -X\tfrac{1}{2}\delta(\Theta/mX).$$

In order that δX should equal $\pm 5 \times 10^{-6}$ at Kew we would require to have

$$\delta(\Theta/mX) = \pm 5 \times 10^{-5}, \text{ approx.}$$

When one uses a suspension sufficient for the collimator magnet alone, without the auxiliary bar, one can with an average magnet get Θ/mX as low as 4×10^{-4} at Kew. Now an error of 10 per cent. in Θ/mX through neglecting the variation of m with temperature or the fluctuations of X at a fixed station is quite out of the question.

On the other hand, suspensions such as are frequently used to carry the auxiliary bar as well as the magnet, and are intended to stand a good deal of rough handling, *may* make Θ/mX at Kew as large as 25×10^{-4}. In such a case error may occasionally arise through not discriminating between m' and m.

In practice the most probable sources of error are inaccuracy in the torsion experiment and variation in Θ, owing to variation of moisture, between the vibration and torsion experiments.

§ 35. *Temperature Coefficients.*—In the method of determining temperature coefficients in vogue at Kew the collimator magnet is fixed inside a wooden box, rigidly attached to the pillar which carries the unifilar. The calculation assumes the mirror magnet—which is suspended as in the deflection experiment—to be exactly perpendicular to the collimator; but, owing to the position of the latter being fixed, this is in general only approximately true. Suppose the deflected magnet to make an angle u with the magnetic meridian, and an angle $\frac{1}{2}\pi \pm \psi$, instead of $\pi/2$, with the collimator magnet. The deflecting force F is given in terms of X by the equation

$$F/X = \sin u/\cos\psi.$$

If F alters owing to change of temperature in the collimator magnet, its increment δF and the increments $\delta\psi$, δu in ψ and u are connected by the relation

$$(F+\delta F)/X = \sin(u+\delta u)/\cos(\psi+\delta\psi).$$

But $\delta\psi$ and δu are equal and so, supposing $\delta F/F$ small, we have

$$\delta F/F = (\cot u + \tan\psi)\,\delta u.$$

The assumption, tacitly made in practice, that the magnetic axes are perpendicular is legitimate only when we may neglect

$$\tan\psi/\cot u.$$

The size of the error varies with the moment of the deflecting magnet, and also with the secular change of declination. A slight change is, I think, desirable either in the experimental arrangements or in the calculation.*

§ 36. A more strictly physical objection to the temperature experiment is that in it the changes of temperature are very sudden, whereas in actual use they are gradual. I am not aware of any experiments bearing directly on the question whether a change of from 15° C. to 40° C, occupying only a minute or two, has the same *temporary* effect on a magnet as an equal change occupying as many hours. Theoretically it would be very desirable to make the changes of temperature slow, but in practice this is troublesome, owing to the disturbing influence of changes in declination and horizontal force. A change of 18° C. in the temperature of a collimator magnet at Kew seldom alters the azimuth of the deflected magnet by more than 10′, so that it is important to avoid the risk of sensible disturbances so far as possible, even though corrections are applied from the magnetic curves.

Another objection is that sudden changes of temperature are apt, like mechanical shocks, to permanently diminish the magnetic moment. It is clear that unless such permanent loss is small, satisfactory values of q and q' are unlikely to result from a calculation which assumes that the temperature effects are purely temporary.

In the normal temperature experiment at Kew the cycle "hot," "mean," "cold" is repeated three times, and the arithmetic mean of the three readings of the deflected magnet's azimuth, answering, say, to the three "hots" is attributed to the temperature which is the arithmetic mean of the three observed temperatures. This method probably eliminates to a considerable extent the influence of permanent changes, supposing them to exist, but it also renders their detection somewhat difficult. To investigate the matter I took a number of temperature experiments, and employing the values found for q and q' calculated

* November 7.—Since this was written the small necessary change in the apparatus has been effected.

the values of the magnetic moment, reduced to a common temperature, at each of the three "hot," "mean," and "cold" readings. Thus, for instance, if the three "hot" temperatures were 35°, 36°, and 37°, the observed magnetic moments were reduced to the mean temperature 36°. The departures from the mean temperatures in this case are so small that trifling errors in q or q' are immaterial. Corrections were carefully applied in all cases from the magnetograms for changes in the horizontal force and declination.

In all, sixteen experiments made on ten different magnets were dealt with; the mean results are given in Table XII. By $(-\delta m/m)$ is meant the diminution found in the magnetic moment divided by the original value of the moment at atmospheric temperature. This is multiplied by 10^6 to avoid decimals.

Table XII.—Mean Values of $(-\delta m/m) \times 10^6$.

Temperature.	2nd reading − 1st.	3rd reading − 2nd.	3rd reading − 1st.
"Hot"	266	95	383
"Mean"	260	75	346
"Cold"	166	56	229
Mean of 3 stages...	231	75	319

In one case only one cycle was taken.

One cannot measure m accurately to one part in 100,000, much less to one part in a million, so that a high degree of accuracy can hardly be claimed for the above figures. It seems, however, perfectly clear that normally there is a loss of magnetic moment, but that at the same time the average loss is very small.

These conclusions were supported by a large number of other cases which I examined, though less carefully.

On the average, according to Table XII, a complete temperature experiment causes a loss of about 0·04 per cent. in the value of m, and of this loss much the greater part occurs during the first temperature cycle.* The true reversible temperature variation of m during a cycle averages very sensibly over 1 per cent.; thus the shock effect can cause but little error in the values of q and q' in the average magnet.

Cases occur, however, in which the loss of magnetic moment is very considerable, and the values of q and q' may have suffered in consequence. The property seems to depend on the tempering, not on the chemical character of the steel; for various magnets which have been rejected for defective retentiveness have behaved normally after further treatment by the maker.

In some of the experiments summarized in Table XII there was a

* It is intended in future to have four temperature cycles, discarding the results of the first cycle.

slight apparent increase of moment at one or more of the stages. In fact, in an old magnet not recently remagnetised, and in a new magnet which had been put through a previous temperature experiment only three days before, the moment appeared greater on the third occurrence of the "hot" temperature than on the first.

In investigating these permanent changes of magnetic moment I received valuable assistance from Mr. R. S. Whipple, then assistant at Kew Observatory.

§ 37. There are several other sources of uncertainty affecting temperature coefficients. For instance, a small fall of temperature may not always have an effect exactly equal and opposite to that of an equal rise.

Again, it is conceivable that the values of q and q' may depend to some extent on the age or strength of the magnet, and so may alter in course of time. On this point I have some interesting data.

In 1894 three collimator magnets after long absence in India were sent to Kew Observatory to be reported on. The opportunity was taken to redetermine their temperature coefficients—originally determined in 1865—before sending them for repair to the maker. After their return they were remagnetised and tested as usual.

Table XIII gives the values deduced for

$$-\frac{1}{m}\frac{dm}{dt} \equiv q + 2q't$$

at 0°, 15°, and 30° C., employing the values found for q and q'.

Table XIII.

Date.	1865.			1894.			1895.		
Temperature.	0°	15°	30°	0°	15°	30°	0°	15°	30°
Magnet i........	240	264	288	252	285	318	347	363	380
,, ii........	219	234	250	158	200	243	—	—	—
,, iii........	290	359	429	327	366	405	363	407	451
Mean for all......	250	286	322	246	284	322	—	—	—
,, ,, i and iii.	265	312	359	289	326	362	355	385	415

Table XIV gives particulars as to the values of the moments of these magnets at the times of the temperature experiments, and the loss of moment which occurred between 1865 and 1894.

The numbering of the magnets is arbitrary, but is the same in both tables. Magnet ii was discarded in 1894, being slightly chipped. It will be seen that the agreement between the mean temperature effects in 1865 and 1894 could hardly be better, notwithstanding the large diminution in the magnetic moments.

Table XIV.

Magnet	Moment in 1865.	Moment in 1894.	Per cent. loss in 29 years.	Moment in 1895.
i	936	568	39	863
ii	776	464	40	—
iii	945	718	24	855

On the other hand, the values found for q and q' in 1895—values based in each case on two consistent experiments—are unmistakably higher than those found in 1865 and 1894. There is no apparent reason for this except the fact that the magnets had been remagnetised. The makers, in reply to inquiries, stated explicitly that "the magnets . . . were only cleaned."

We seem driven to the conclusion that whilst a gradual loss of magnetic moment *may* not appreciably influence the values of the temperature coefficients, the remagnetisation of a magnet does influence these coefficients, in at least some cases.

§ 38. *Induction Coefficient.*—The method of determining μ has been already described in § 3. There are here also several uncertainties.

The assumption that the relation between the temporary moment and the field is strictly linear may not be sufficiently exact; 0·44 C.G.S. unit is a considerably stronger field than Lord Rayleigh found to limit the linear part of his induction curves, and recent German observers seem disposed to narrow his limits. The accuracy of the assumption is doubtless less in some collimators than in others, but I have no data on which to proceed.

*Again, it is open to doubt whether a magnet possessed of a large permanent moment responds equally to the action of two small fields, one tending to increase, the other tending to diminish, the total moment. This uncertainty enters into the application of μ as well as into its original determination. For in the vibration experiment the temporary and permanent magnetisms are in the same sense, whereas in the deflection experiment they are in opposite senses.

If the presence of a permanent magnetic moment influences the value of μ, then the value found at Kew, even if otherwise above criticism, will presumably become less exact as the moment falls off.

Another source of uncertainty is the probability that μ varies with the temperature.

We can best judge of the possible influence of these sources of uncertainty by reference to the formula

* Lamont, 'Handbuch des Erdmagnetismus,' pp. 149—150, says μ is less for strengthening than for weakening fields, especially when the permanent moment is very large.

$$\delta X/X = -\delta\mu r^{-3} \operatorname{cosec} u - \delta\mu' r^{-3},$$

connecting the error δX in X with the departures $\delta\mu$ and $\delta\mu'$ of μ from its assumed value during the vibration and deflection experiments.

As cosec u exceeds 1, we see that if $\delta\mu$ and $\delta\mu'$ be equal, the first term on the right of the equation is the larger of the two; thus an error of given amount in μ has most effect in the vibration experiment.

We have approximately

$$r^{-3} \operatorname{cosec} u = X/2m,$$

and putting $r = 30$, we should find for an average collimator magnet at Kew

$$\delta X = -10^{-6} (20\delta\mu + 7\delta\mu'), \text{ approx.}$$

The average value of μ in Table I is 5·85; thus if $\delta\mu$ and $\delta\mu'$ exceeded 3 per cent. of the value of μ the consequent error in X would become sensible.

A change of 30° C. in temperature seldom alters m by more than 2 per cent., so in temperate regions the neglect of temperature variation in μ seems hardly likely to lead to error, unless of course temporary magnetism is more susceptible to temperature changes than permanent magnetism.

In tropical countries the neglect is likely to be more serious.

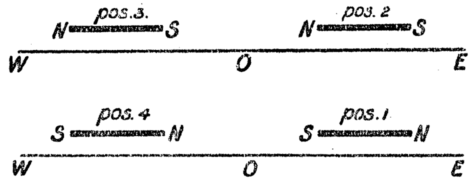
[Since this paper was written, Professor Mascart, in 'Terrestrial Magnetism,' has called attention to a source of uncertainty which I had supposed negligible. He points out that in the vibration experiment the proper induction coefficient is really $\mu - \nu$, where μ has the meaning attached to it here, while ν is the coefficient of induction for a magnetic field perpendicular to the magnet's length. Professor Mascart describes a method of determining experimentally μ, ν, and μ'—the coefficient when temporary and permanent longitudinal magnetisations are opposed. The method is, he says, easily applied and the results consistent; but the apparatus required seems a little complicated. Professor Mascart quotes no numerical results.

In the Kew pattern collimator magnet I should anticipate ν/μ to be small, but it may not be negligible. Experimental investigation of the point is certainly desirable.]

Asymmetry in Magnets.

§ 39. In the deflection experiment it is customary to describe the collimator magnet as being on the *east* or *west* side of the mirror magnet, and as having its north pole *east* or *west.* The directions are not strictly east and west, but perpendicular to the mirror magnet, as deflected out of the magnetic meridian. As the sense intended, however, is quite clear, I shall employ the usual terminology.

In the accompanying figure, which is not drawn to scale, W and E represent the "west" and "east" ends of the graduated bar on which



slides the frame carrying the collimator magnet. The graduations run in both directions from O, the centre of the bar. The centre of the mirror magnet should be on the vertical line through O, and also on the horizontal line which is the prolongation of the magnetic axis of the collimator NS. The figure shows the collimator magnet in the four positions which it occupies during the deflections made at one of the two distances, say 30 cm. Supposing a 30 cm. deflection to be the first of the day, the four positions are assumed in the order 1, 2, 3, 4. It is the rule at Kew Observatory that if a 30 cm. observation takes precedence on a given occasion, then a 40 cm. observation has precedence on the next. On the second occasion the four 30 cm. positions occur in the order 2, 1, 4, 3. The object is very probably to eliminate so far as possible the influence of changes of force or temperature on the value of P, which is calculated from a large number of observations.

§ 40. If there were perfect symmetry one should have identical readings in positions 1 and 4, and again in positions 2 and 3. But, in reality, agreement in these readings is wholly exceptional. To eliminate the disturbing influence of observational errors and changes in force a number of deflection experiments are necessary. Thus, the only case in which I have had the data requisite for an exhaustive study is that of the Kew unifilar itself.

In this instance the phenomena seem adequately explained by the hypothesis of the existence of such asymmetry as we should anticipate *à priori*.

It is doubtless the aim of the maker to fit the suspension tube so that the centre of the mirror magnet and the centre, or zero, of the graduated bar in the deflection experiment shall lie in a vertical plane perpendicular to the bar; but complete success is unlikely to attend his attempts. Accordingly, my first assumption is that the centre of the mirror magnet is really at a distance z, counted positive towards the "east," from the vertical plane perpendicular to the bar through the centre O.

The frame carrying the collimator magnet has a fiducial mark at the centre of its edge, which is set during the deflection experiment at the

division 30 or 40 cm. on the bar. From the centre of the upper edge of the frame a pillar extends vertically upwards, so as to fit into the hollow shank which forms part of the appendages of the collimator magnet. The vertical plane perpendicular to the bar through the fiducial mark should contain the centre of the vertical pillar and also that of the magnet. Supposing the frame to have fiducial marks on both sides, and that these agree, one can test the existence of the first possible cause of asymmetry by turning the frame end for end, keeping the direction of the magnet unchanged; the azimuth of the mirror magnet will serve as a criterion. This kind of asymmetry has not been detected in the Kew unifilar; and if existent its influence would be nearly eliminated in a large number of observations in which either position of the sliding frame occurred promiscuously. The existence of the second species of asymmetry is less easily tested, but is more probable *à priori.* Accordingly my second assumption is that during the deflection experiments the centre of the magnet is at a distance y from the vertical plane which passes through the fiducial mark, and is perpendicular to the magnet's length. Here y is counted positive when the magnet's centre and north pole are on the same side of the fiducial mark.

§ 41. On these two hypotheses it is obvious from fig. 1, that when the apparent distance between the centres of the magnets is r, the true distances are as follows :—

Position.	1.	2.	3.	4.
True distance	$r+y-z$	$r-y-z$	$r+y+z$	$r-y+z$.

Now suppose that u and $u+\delta u$ are the deflection angles of the mirror magnet answering to the true distances r and $r+\delta r$. Then, according to the first approximation formula,

$$(r+\delta r)^3 \sin(u+\delta u) = r^3 \sin u\,;$$

whence, neglecting squares and products of δu and δr, we have

$$\delta u = -3r^{-1} \tan u\, \delta r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10).$$

In what follows I employ u when $r = 30$ cm., and u' when $r = 40$ cm., and distinguish the several positions of fig. 1 by suffixes.

Referring to the scheme of values of δr given above, we find—

Position 1.

$$\delta u_1 = -\tfrac{1}{10}(y-z)\tan u,$$

$$\delta u'_1 = -\tfrac{3}{40}(y-z)\tan u'.$$

Position 2.

$$\delta u_2 = \tfrac{1}{10}(y+z)\tan u,$$

$$\delta u'_2 = \tfrac{3}{40}(y+z)\tan u'.$$

Position 3.

$$\delta u_3 = -\tfrac{1}{10}(y+z)\tan u,$$

$$\delta u'_3 = -\tfrac{3}{40}(y+z)\tan u'.$$

Position 4.

$$\delta u_4 = \tfrac{1}{10}(y-z)\tan u,$$

$$\delta u'_4 = \tfrac{3}{40}(y-z)\tan u'.$$

Whence

$$\delta u_4 - \delta u_1 = \tfrac{1}{5}(y-z)\tan u, \qquad \delta u_2 - \delta u_3 = \tfrac{1}{5}(y+z)\tan u,$$

$$\delta u'_4 - \delta u'_1 = \tfrac{3}{20}(y-z)\tan u', \qquad \delta u'_2 - \delta u'_3 = \tfrac{3}{20}(y+z)\tan u'.$$

The mean results from the thirty-nine absolute determinations of horizontal force made with the Kew unifilar in 1892, were as follows :—

$\delta u_4 - \delta u_1$	$\delta u_2 - \delta u_3$	$\delta u'_4 - \delta u'_1$	$\delta u'_2 - \delta u'_3$.
10′ 22″.	27′ 29″.	3′ 1″.	8′ 25″.

Also, approximately,

$$u = 15^\circ\ 30', \qquad u' = 6^\circ\ 30'.$$

Thence we have—

From observations at 30 cm.	From observations at 40 cm.
$y+z = 5 \times 3{\cdot}606 \times 0{\cdot}00799 = 0{\cdot}144$ cm.,	$y+z = 6{\cdot}\dot{6} \times 8{\cdot}777 \times 0{\cdot}00245 = 0{\cdot}143$ cm.,
$y-z = 5 \times 3{\cdot}606 \times 0{\cdot}00302 = 0{\cdot}054$,,	$y-z = 6{\cdot}\dot{6} \times 8{\cdot}777 \times 0{\cdot}00088 = 0{\cdot}052$,,
whence	whence
$y = 0{\cdot}099$ cm. $z = 0{\cdot}045$ cm.	$y = 0{\cdot}098$ cm. $z = 0{\cdot}045$ cm.

If the hypotheses on which the calculations are based are incorrect, the agreement between the two sets of values found for y and z is certainly remarkable.

I repeated the calculations for the data from the same unifilar in 1893, and again the two sets of values for y and z were in excellent agreement, the means being

$$y = 0{\cdot}095 \text{ cm.}, \qquad z = 0{\cdot}039 \text{ cm.}$$

§ 42. *Consequences of Asymmetry.*—Using the first approximation formula, we have of course when we neglect δr^2 and δu^2

$$\delta u_1 + \delta u_2 + \delta u_3 + \delta u_4 = 0.$$

Thus to estimate the size of the error we must go as far as squares and products of small quantities, replacing (10) by

$$\delta u = -3r^{-1}\tan u\,\delta r + \tfrac{1}{2}\tan u\,\delta u^2 - 3r^{-1}\delta u\,\delta r - 3r^{-2}\tan u\,\delta r^2 \ldots \quad (11).$$

Substituting the first approximation value for $\delta u/\delta r$ in the small terms in the usual way, we find

$$\delta u = -3r^{-1}\tan u\,\delta r + \tfrac{3}{2}\tan u\,(3\tan^2 u + 4)\,r^{-2}\delta r^2 \ldots\ldots \quad (12).$$

Taking $r = 30$, and ascribing to δr in succession the values corresponding to the four positions, I find on reduction

$$\delta\bar{u} \equiv \tfrac{1}{4}(\delta u_1 + \delta u_2 + \delta u_3 + \delta u_4) = \tfrac{1}{600}\tan u\,(3\tan^2 u + 4)\,(y^2 + z^2) \ \ldots \ (13).*$$

* The method followed above has the advantage of leading directly to the value

Taking the data for the Kew unifilar in 1892, we find

$$\delta\bar{u} = 23 \times 10^{-6} \text{ (or about } 48''\text{)}.$$

Now $\delta\bar{u}$ is the error, in circular measure, in the observed deflection u, and the consequent error in the horizontal force, deduced from the relation

$$X^2 \sin u = \text{constant},^*$$

is
$$\delta X = -\tfrac{1}{2} X \cot u \delta\bar{u} \quad \ldots\ldots\ldots\ldots \quad (14).$$

In the case of the Kew unifilar, we have approximately

$$X = 0{\cdot}183, \qquad \cot u = 3{\cdot}61,$$

whence
$$\delta X = -0{\cdot}0000075 \text{ C.G.S., approx.}$$

Thus the error is nearly one unit in the last significant figure. As the error varies as $y^2 + z^2$, it would become of very sensible magnitude if y and z were twice or thrice as large as in the Kew unifilar. If I may judge from the figures recorded of unifilars under examination at the Observatory, such large values of y and z really exist.

The source of error just considered alters the observed deflections at 30 and 40 cm. in the same sense, and seems unlikely to exert an appreciable influence on the value of P.

§ 43. *Causes of Asymmetry.*—A collimator magnet ought to be adjusted in its stirrup until it is exactly horizontal in the vibration experiment. The magnetic forces acting on it tend, in this magnetic hemisphere, to pull the north pole down, and consequently the centre of gravity of the composite body composed of magnet and appendages must be out of the vertical, which is the prolongation of the suspending fibre, and on the same side of it as the south pole. If the stirrup is symmetrical, and if the lens and scale of the magnet are equal in weight and symmetrically situated, then the C.G. of the magnet itself must, in this hemisphere, lie on the same side of the suspending fibre as the south pole.

Now the suspending fibre in the vibration experiment, if produced, should cut the magnetic axis at the same point as it is cut in the deflection experiment by the vertical plane which is perpendicular to the magnet, and contains the fiducial mark on the sliding frame.

Thus in a perfectly symmetrical magnet the middle point of the magnet's length, and so the middle point of the line joining the "poles" (assuming magnetic symmetry) must lie on one side of the fiducial mark, and so of the graduation on the bar which coincides with it.

Calling the error thus introduced in the distance r between the

of $\delta\bar{u}$, a quantity whose physical significance is easily grasped. But a sufficiently exact value of δX can be obtained more simply from the relation $(r + \delta r)^3 (X + \delta X)^2 = r^3 X^2$ (see (4)).

magnets' centres y, measured as in § 40, we can find its value if we know the circumstances of the case. Thus suppose the magnet to have a moment 840 and to weigh, with lens and scale, 30 grams; then at a place where g (gravity) is 980, and the vertical magnetic force is 0·44, we have

$$-y = (840 \times 0{\cdot}44) \div (980 \times 30) = 0{\cdot}013 \text{ cm., approx.}$$

The value thus found for y is not only much less numerically, but even opposite in sign to the value calculated for the Kew collimator magnet in § 41. There is nothing surprising in this, because the perfect symmetry which our last calculation assumes in the magnet and its appendages is a remote probability. For instance, the lens and scale of the magnet, in the only case where I had them weighed, differed by nearly half a gram. If other things were symmetrical this alone would remove the C.G. nearly 0·08 cm. from the symmetrical position.

Mechanical asymmetry might arise in many other ways. The magnet itself, for instance, might be slightly conical.

It is also at least conceivable that magnetic asymmetry may exist. If a magnet were conical, or had one end tempered differently from the other, I see no reason to expect its "poles" to be equidistant from the middle point of its length.

§ 44. Whilst numerous causes may contribute to the asymmetrical displacement denoted by y, the first mentioned appears the most interesting. Not merely is it, as we have seen, unavoidable, but it varies if either m or the vertical force alters.

The vertical force on the earth's surface varies from about + 0·6 to − 0·6 C.G.S. unit. Thus if the typical magnet above considered were adjusted so as to be horizontal under the one limiting force, it would have to be shifted about 0·034 cm. in its stirrup to become horizontal under the other.

The shifting in the stirrup entailed by this variable cause of asymmetry slightly affects the moment of inertia. For instance if the symmetrical magnet considered above, for which $y = 0{\cdot}013$ cm. at Kew, were used in a tropical station where the vertical force vanished, y should be reduced to 0, and the consequent change in moment of inertia would be

$$\delta K = -30\,(0{\cdot}013)^2 = -0{\cdot}005 \text{ C.G.S. unit, approx.}$$

The mean K in Table I is $2711/\pi^2$, or 274 approximately; and with these values

$$\delta K/K = -2 \times 10^{-5}, \text{ approx.}$$

Supposing no allowance made for this, the consequent error in X would be

$$\delta X = -X \times 10^{-5}.$$

This would hardly be sensible supposing X measured as usual to five significant figures. It might happen however that owing to asymmetry in the stirrup the C.G. of the magnet—with scale and lens—had to lie, in the absence of vertical force, at a distance d, comparable with 1 mm. on one side of the suspension fibre. Under such circumstances the change in the moment of inertia when the vertical force altered from 0·44 to 0 would exceed that calculated above in the ratio

$$2d : 0{\cdot}013.$$

As perfect mechanical symmetry must be the exception, we see that this source of uncertainty is likely to be considerably more serious than might be supposed from our first example.

The above considerations may suggest that shifting the collimator magnet in its stirrup is a mistake. If, however, this is not done when required to keep the magnet horizontal, the magnet and appendages must tilt over slightly. This not only alters the moment of inertia, but likewise prevents the magnetic couple in the vibration experiment from having its proper value.

The source of trouble we have just been considering is distinctly exceptional in this respect, that it is most serious in magnets of large magnetic moment.

Law of Action between Magnets.

§ 45. The formula

$$\text{couple} = 2mm'r^{-3}(1 + \mathrm{P}r^{-2} + \mathrm{Q}r^{-4} + \ldots\ldots)$$

assumes that the centre of the mirror magnet lies on the prolongation of the axis of the collimator magnet, and that the axes of the two magnets are perpendicular to one another. With the magnets in the position supposed, it is not difficult to calculate the values of P and Q, if we can regard a magnet as consisting of two equal and opposite "poles," the product of whose distance into the strength of either constitutes the magnetic moment. Thus if 2λ and $2\lambda'$ be the distances between the poles of the collimator and mirror magnets respectively, I find

$$\text{couple} = 2mm'r^{-3}\{1 + (2\lambda^2 - 3\lambda'^2)r^{-2} + \tfrac{3}{8}(8\lambda^4 - 40\lambda^2\lambda'^2 + 15\lambda'^4)r^{-4} + \ldots\ldots\} \quad \ldots\ldots (15),$$

so that

$$\left.\begin{aligned} \mathrm{P} &= 2\lambda^2 - 3\lambda'^2 \\ \mathrm{Q} &= 3(8\lambda^4 - 40\lambda^2\lambda'^2 + 15\lambda'^4)/8 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (16).$$

When $r = 30$ cm.

$$\left.\begin{aligned} \mathrm{P}r^{-2} &= (2\lambda^2 - 3\lambda'^2)/900 \\ \mathrm{Q}r^{-4} &= (8\lambda^4 - 40\lambda^2\lambda'^2 + 15\lambda'^4)\,10^{-4}/216 \end{aligned}\right\} \quad \ldots\ldots\ldots (17).$$

We have $P = 0$ when $\lambda'/\lambda = \sqrt{2/3} = 0{\cdot}8165$, approx.,

$Q = 0$ „ $\lambda'/\lambda = 0{\cdot}467$, approx.

We can thus make either P or Q vanish, but not both simultaneously.

When

$$P = 0,$$
$$Q = -9\lambda^4/2.$$

§ 46. Both collimator and mirror magnets are comparatively "short," *i.e.*, their lengths are only about ten times their diameters. In such magnets we should expect, according to Coulomb,

$$2\lambda = 2l/3, \qquad 2\lambda' = 2l'/3,$$

where l and l' are the lengths of the magnets. The formulæ by Green, Jamin, and others are more complicated.

Perhaps all we are justified in saying, *à priori*, is that λ and λ' cannot exceed $l/2$ and $l'/2$, and that the collimator and mirror magnets are sufficiently similar in pattern to make it probable that the assumption

$$\lambda'/\lambda = l'/l$$

is not far wrong.

To throw some light on the question, I had measurements made of the magnets of the Kew unifilar and of a unifilar of class E. The Kew unifilar belongs to group B, and its P is negative but exceptionally small. The instruments of group E are, as we have already seen, exceptionally uniform in character, and invariably have their P large and positive.

The results were as follows, lengths being in centimetres,

Unifilar.	l.	l'.	l'/l.
Kew, group B	9·35	7·60	0·81
„ E	9·17	6·35	0·69

Supposing $\lambda'/\lambda = l'/l$, we should have in these two cases

λ'/λ.	P/λ^2.	Q/λ^4.
0·81	0·00	−4·4
0·69	+0·57	−2·9

These figures, taken in conjunction with our previous data, are on the whole distinctly favourable to the hypothesis

$$2\lambda/l = 2\lambda'/l' = n \text{ (a constant)} \quad \ldots\ldots\ldots\ldots\ldots \quad (18)$$

Accepting this hypothesis provisonally, I had the curiosity to see what value we should obtain for n by ascribing to P the mean value found for unifilars of group E, taking for l and l' the values quoted above for a unifilar of that group.

Equating to one another the two values of Pr^{-2} at 30 cm., the one as given above, the other as given in Table I, we have

$$0{\cdot}57\lambda^2 \times 30^{-2} = 655 \times 10^{-5}.$$

Thus $$\lambda = 3{\cdot}21, \text{ approx.},$$

and so $$n \equiv 2\lambda/l$$
$$= 6{\cdot}24 \div 9{\cdot}17 = 0{\cdot}67, \text{ approx.}$$

The practically exact agreement with Coulomb's relation

$$2\lambda = 2l/3$$

seems to warrant the conclusion that if we employ this relation in calculating Q the result is likely to be of at least the right order of magnitude.

On this hypothesis, taking the values 9·35 and 9·17 for l, as fairly representative of the two groups, we have with $r = 30$ cm.,

Probable value of Qr^{-4} in unifilar of group B $= -5 \times 10^{-4}$, approx.
" " " E $= -3 \times 10^{-4}$ "

Assuming the value of P to be correctly determined, the error in X due to the omission of the Q term is given by

$$\delta X/X = -\tfrac{1}{2}Q/30^4.$$

Taking the numerical values found above for Q, we should have when $X = 0{\cdot}18$—

For mean unifilar of group B, $\delta X = +5 \times 10^{-5}$, approx.
" " " E, $\delta X = +3 \times 10^{-5}$, "

There is admittedly much uncertainty in these numerical estimates, but they undoubtedly indicate that the neglect of Q requires justification.

If the Q term is not negligible then the ordinary method, which assumes that the P term is the sole cause of the difference between the two values of m/X, given by the first approximation formula with $r = 30$ and $r = 40$, must lead to a wrong value for P.

Now the P correction does not strike a mean between the results at 30 and 40 cm., but adds to or subtracts from *both* values of m/X. Thus the indirect consequences of neglecting Q may be as important as the direct.

[Lamont, in his 'Handbuch des Erdmagnetismus' (Berlin 1849), discusses the general case of the action of one magnet on another, the distribution of "free" magnetism being arbitrary. The results (16) for

P and Q, though apparently not given explicitly by Lamont, could be easily deduced from his general formulæ, see especially his pp. 44 and 45. Dr. C. Borgen, in 'Terrestrial Magnetism,' October, 1896, pp. 176–190, carries Lamont's formulæ somewhat farther. The position taken by the two magnets in the deflection experiment is that termed by Borgen "Erste Lamont'sche Hauptlage." According to Lamont's formulæ, the relative importance of Q, and the values of l'/l, for which P and Q vanish, must vary according to the law of distribution of free magnetism. At the same time, it would appear that so long as the law of distribution is the same for the two magnets, the conditions for P vanishing is hardly likely to depart much from

$$l'/l = \sqrt{2/3}.$$

The idea adopted in the text of assuming "poles," and proceeding on the hypothesis

$$2\lambda/l = \text{constant},$$

is one of which Lamont says on his p. 45, "Dies ist ein sehr nützlicher Mittel um approximativ den Werth der höheren Glieder zu bestimmen . . . ." Thus, though not new, as I had supposed, it has the advantage of being recommended by one of the leading pioneers of terrestrial magnetism.]

§ 47. The exact symmetry in the position of the two magnets which is postulated in obtaining the formula for the deflection is rather to be hoped for than expected. For instance, the magnetic centre of the mirror magnet is hardly likely to lie exactly on the prolongation of the magnetic axis of the collimator magnet, nor is the magnetic axis of the mirror magnet likely to be exactly normal to the mirror. I have, however, no experimental data bearing on either of these points.

§ 48. The present paper may seem premature, in view of the number of questions which it raises without finally answering. To wait, however, until experiment had given a final answer to all the questions would simply mean shelving the whole subject indefinitely. Many of the questions could be satisfactorily dealt with only by elaborate experiments. Such experiments are hardly likely to be carried out at any public institution, within a reasonable time, until the necessity for them has been clearly demonstrated.

The investigations embodied in this paper have extended over several years, and the results obtained are likely, I think, to be of immediate use.

The unifilar magnetometer is not the only instrument capable of measuring the horizontal force. Induction coil methods have of late years been introduced and advocated by eminent authorities. The question of the relative merits of magnetometers and induction coils is very likely, I think, to come to the front in a few years, and it is hardly

likely to be adequately considered unless the sources of weakness in the respective methods are clearly understood.

Again, there has been an increasing tendency of late years, in most lines of physical research, to add figure to figure, and to aim at higher accuracy, or at least the appearance of higher accuracy. If people were content with giving magnetic force to four significant figures, the majority of the sources of uncertainty specified in this paper need awaken little apprehension. But now-a days hardly any one is content with less than five significant figures, and one occasionally sees six. I am not prepared to say that the retention of six figures is indefensible in the case of open range magnetographs, when it is clearly understood that *differences* only are concerned, but I do think that with unifilars of the Kew pattern, as hitherto made, the least probable error we can reasonably expect in an absolute measurement is two or three units in the fifth significant figure.

§ 49. We have seen that there is every reason to expect that the values of the horizontal force given by a unifilar are in error to a different extent, according to the temperature and the magnitude of the force. This must introduce a source of uncertainty into the comparisons effected between unifilars at distant observatories through the intermediary of a travelling instrument. To adequately forewarn those engaged in such comparisons may be to put them in a position to obtain more satisfactory results.

§ 50. For the opinions expressed in this paper, and for the accuracy of its conclusions, I am alone responsible; but it is only proper that I should acknowledge the valuable assistance given me by Mr. T. W. Baker, Chief Assistant at the Kew Observatory. For many years Mr. Baker has taken the great majority of magnetic force observations at Kew, and his experience in determining the "constants" of collimator magnets is probably quite unique. Mr. Baker has always been ready to place his great practical knowledge of the subject freely at my disposal, and I have also to thank him for carefully carrying out a variety of special experiments, made to elucidate doubtful points at various stages of the inquiry.

"The Absorption of Röntgen's Rays by Aqueous Solutions of Metallic Salts." By the Right Honourable LORD BLYTHSWOOD, LL.D., and E. W. MARCHANT, D.Sc. Communicated by LORD KELVIN, F.R.S. Received March 11,—Read June 15, 1899.

The absorption of X rays by metallic salts is a subject that has not received very much attention up to the present time, although it appears to be of considerable importance. It seemed possible that if the